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Inventory management 2

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PPU426 – HT 2018

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Today's topic

- Inventory measures
- The ABC system
- Specific situations in finding economic order quantity
    - Non-instantaneous replenishment
    - Quantity discount
    - One-period decisions

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Inventory measures

- Average aggregate inventory value (A) — (Example)

  $= \sum_i$ (No of unit X_i * Value of unit X_i) — 10 MSEK

- Week of supply

  = A / Weekly sales

  Weekly sales = 2M

  W of S = 10/2 = 5w

- Inventory turnover

  = Annual sales / A

  Annual sales = 80M

  Inv. turnover = 8 turns

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EXAMPLE 1

Inventory measures

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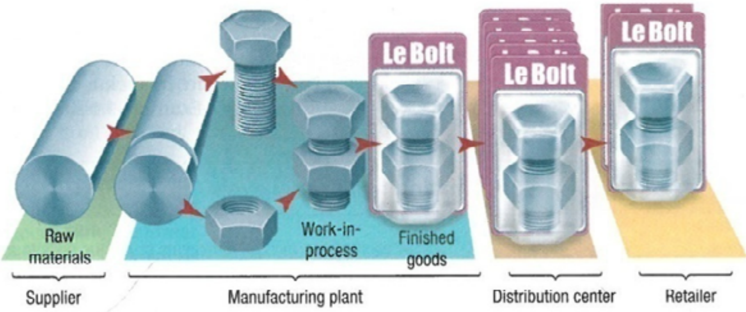


- Annual sales : $3,410,000
- Operates 52 weeks
- Inventory info on the right

Category	Part Number	Average Level	Unit Value
Raw materials	1	15,000	$ 3.00
	2	2,500	5.00
	3	3,000	1.00
Work-in-process	4	5,000	14.00
	5	4,000	18.00
Finished goods	6	2,000	48.00
	7	1,000	62.00

Questions:

a. What is the average aggregate inventory value?
b. Week of supply?
c. Inventory turnover?

a. Average aggregate inventory value (A) ?

Part Number	Average Level		Unit Value		Total Value
1	15,000	×	$ 3.00	=	$ 45,000
2	2,500	×	5.00	=	12,500
3	3,000	×	1.00	=	3,000
4	5,000	×	14.00	=	70,000
5	4,000	×	18.00	=	72,000
6	2,000	×	48.00	=	96,000
7	1,000	×	62.00	=	62,000
	Average aggregate inventory value			=	$360,500

b. Week of supply?

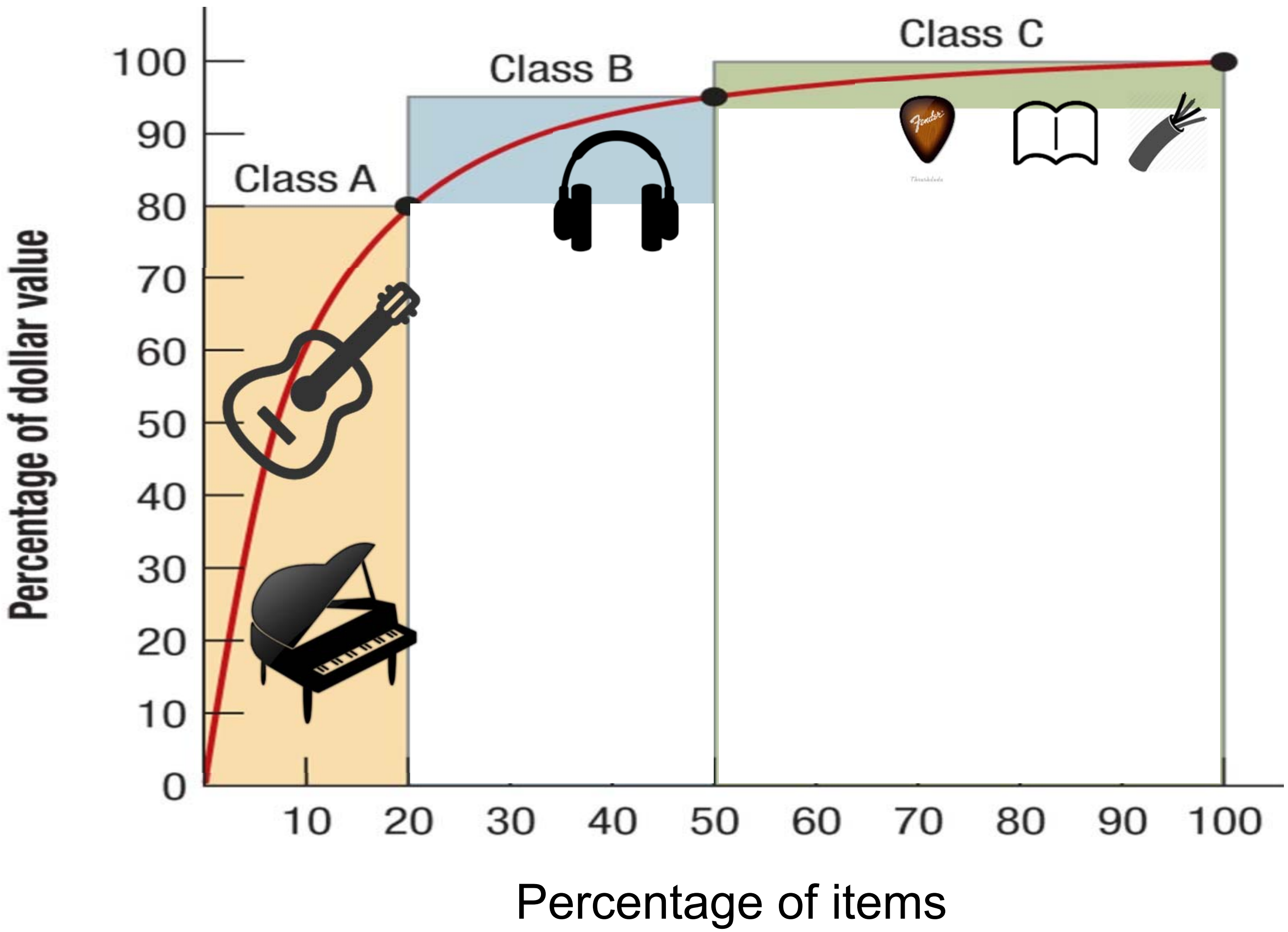
= A / week sales = A / (annual sales/52)

= 360,500 / (3,410,000/ 52) = 5,5 weeks

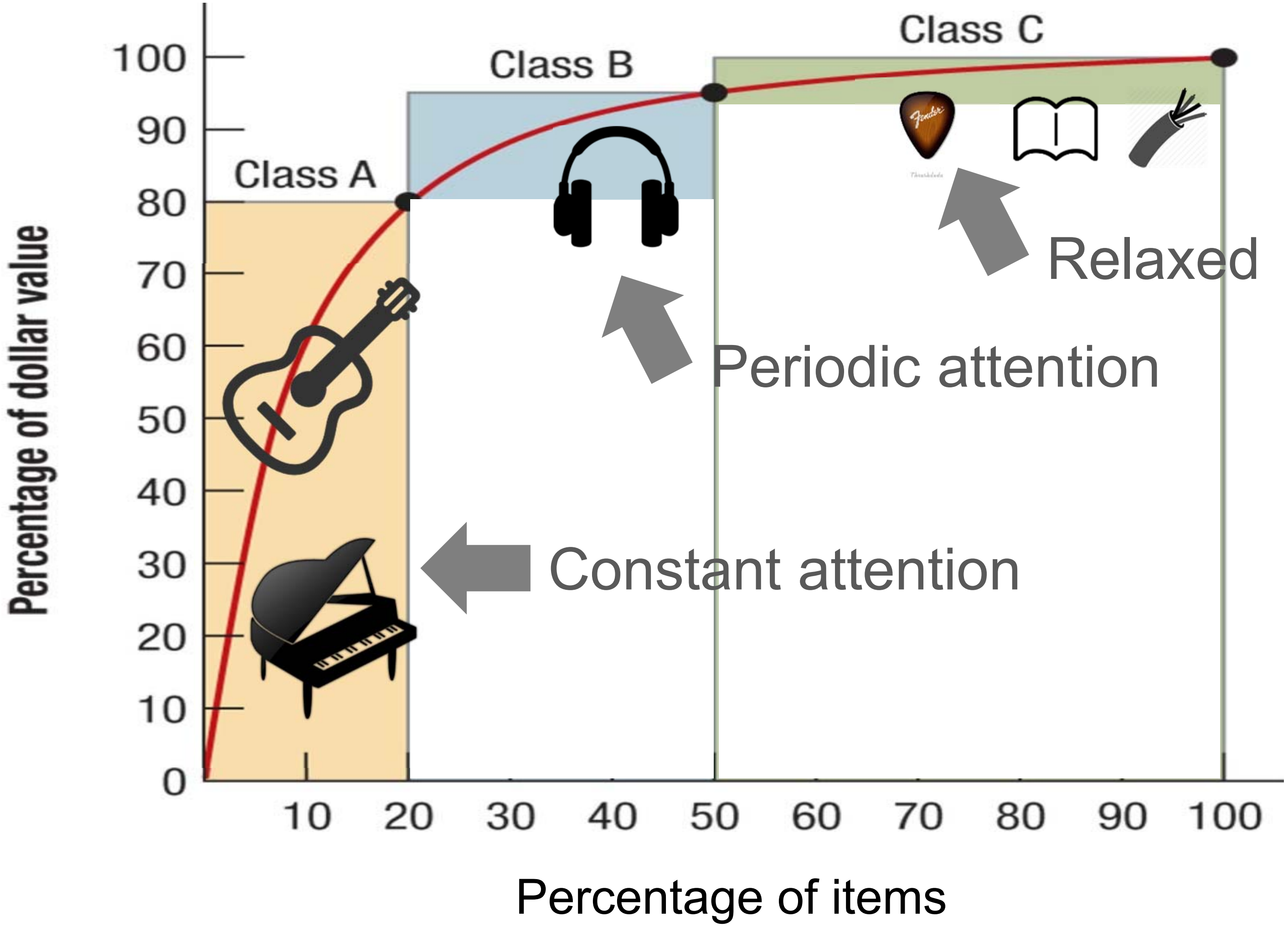
c. Inventory turnover?

= Annual sales / A = 9,5 turns

ABC analysis



ABC analysis



The Pareto principle

The Pareto Principle (also called 80/20 rule)

The 80/20 rule means that in anything, a few (20 percent) are vital and many (80 percent) are less vital.

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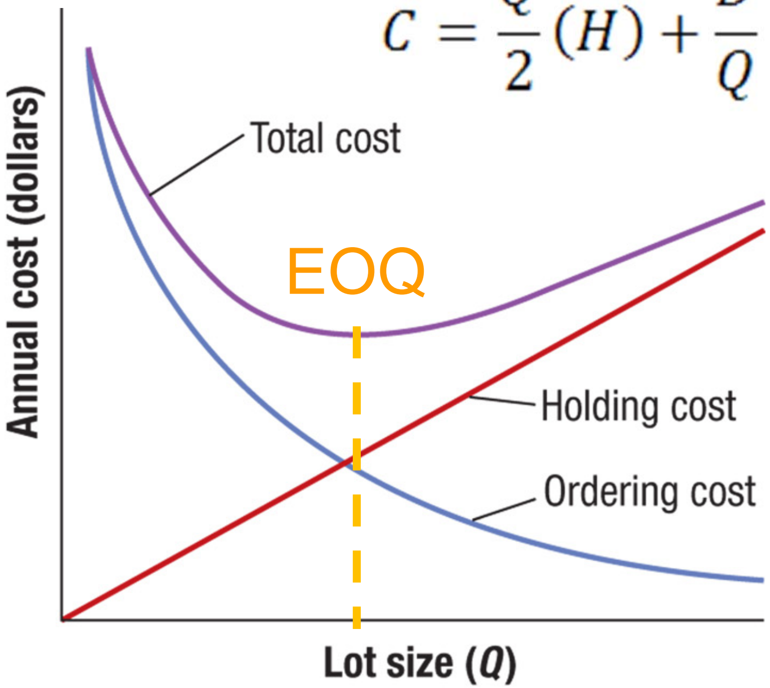
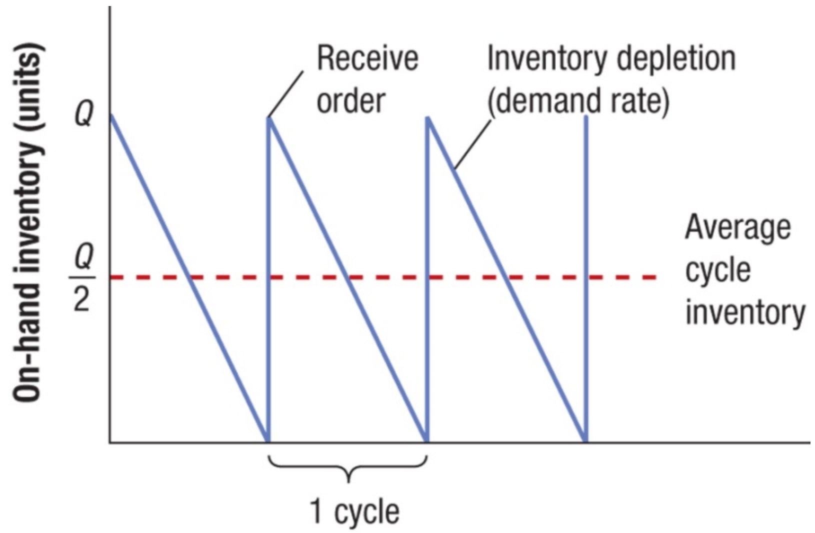
EOQ at standard situation

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$





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Non-instantaneous replenishment

Example : Chemical plant

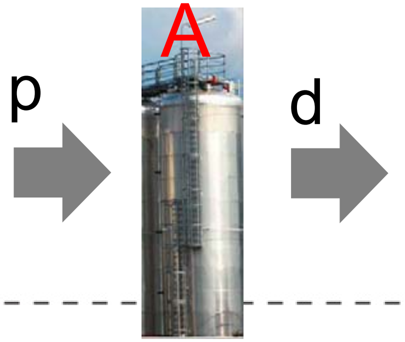
Producing chemical A

Used for another process

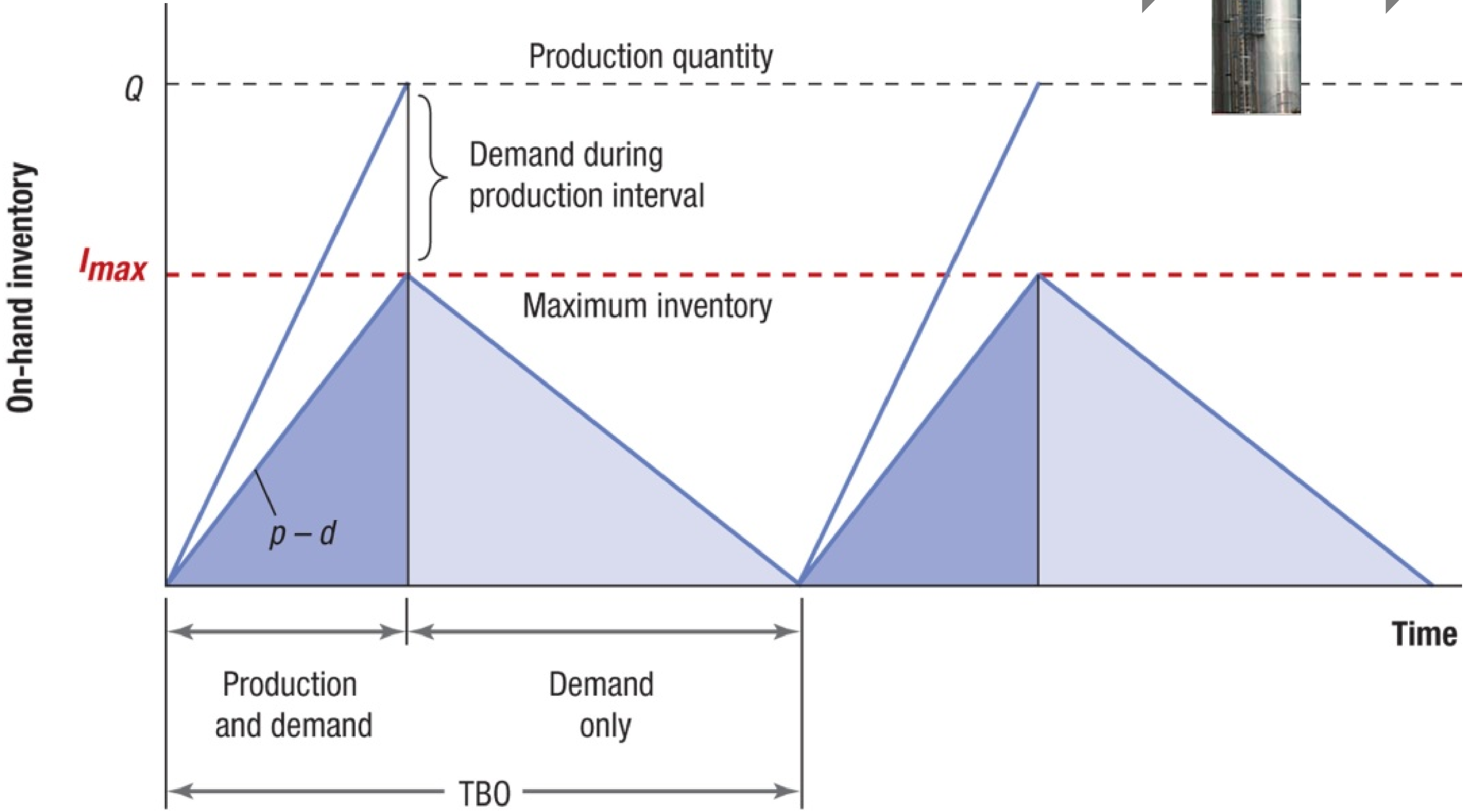
Setup cost to initiate the production

Produce Q when becomes empty

Inventory profile for Non-instantaneous replenishment



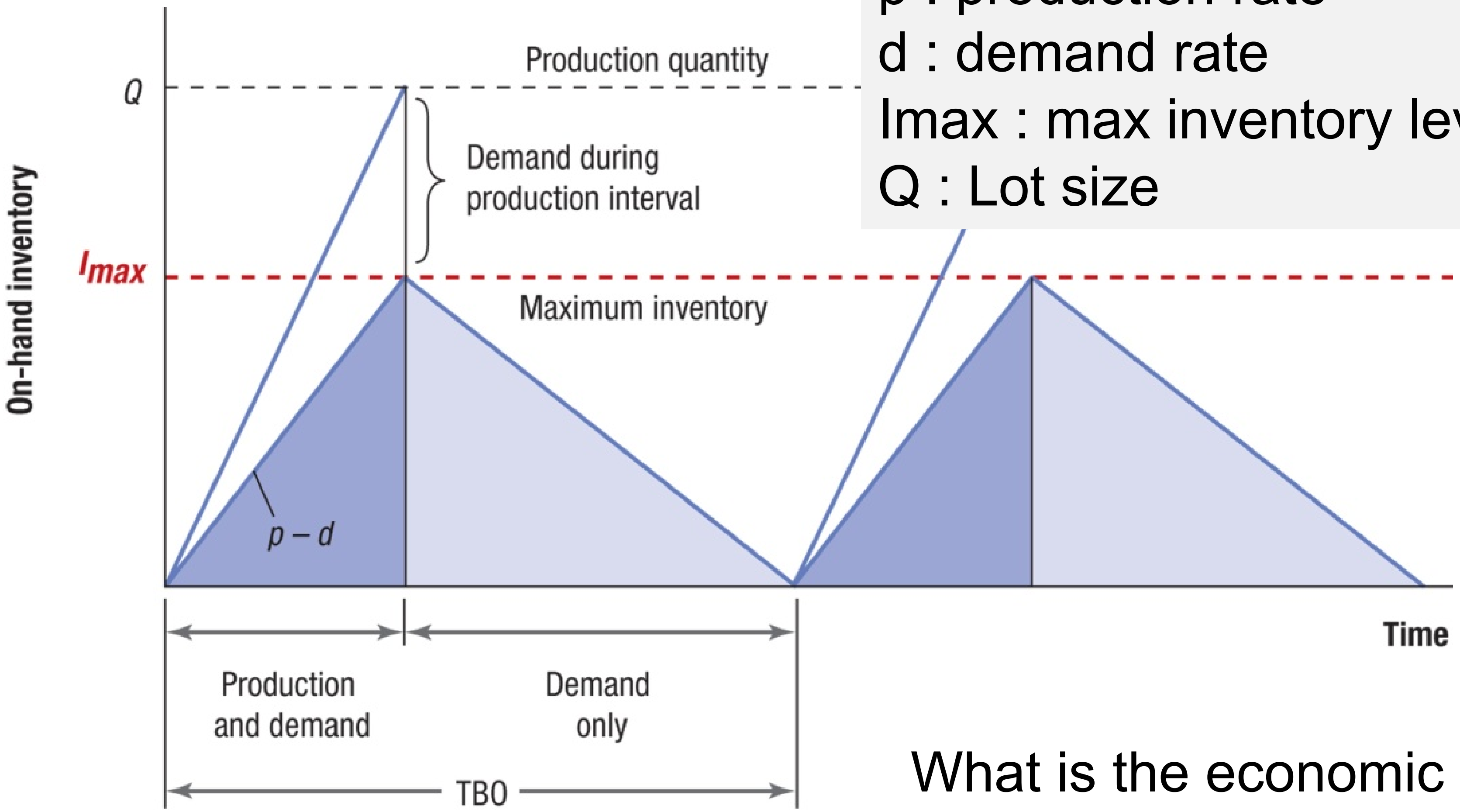
For chemical A



Inventory profile for Non-instantaneous replenishment

For chemical A

p : production rate
d : demand rate
Imax : max inventory level
Q : Lot size



What is the economic Q?

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Non-instantaneous replenishment Finding economic Q

Maximum cycle inventory (I_{max}):

I_{max} = Production time to produce Q x Building up rate

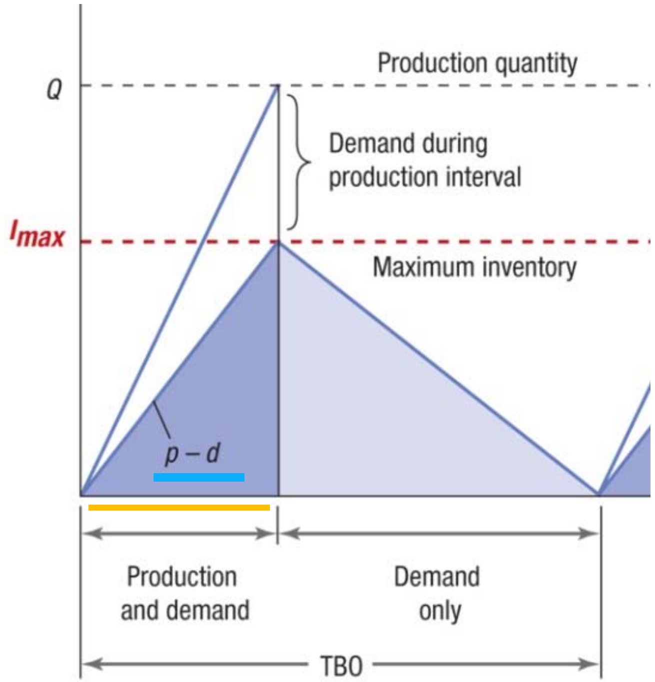
$$I_{max} = \frac{Q}{p}(p-d) = Q\left(\frac{p-d}{p}\right)$$

where

p = production rate

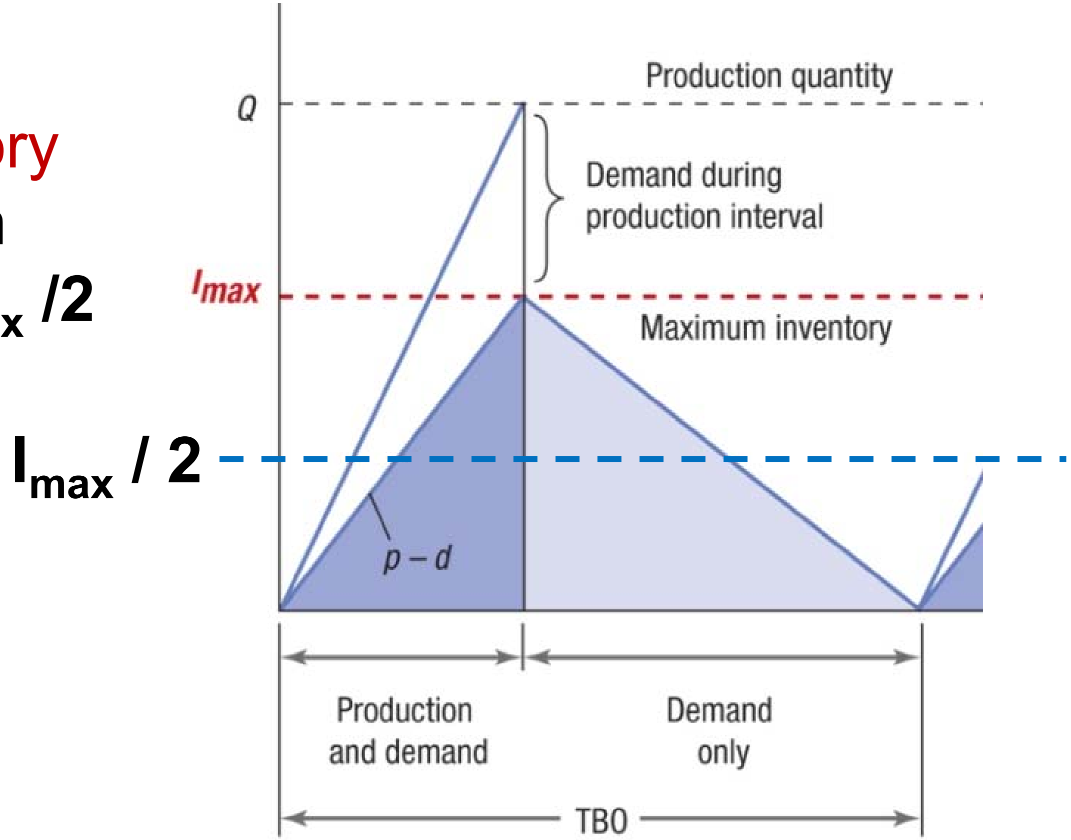
d = demand rate

Q = lot size



Non-instantaneous replenishment
Finding economic Q

Average cycle inventory is no longer Q/2 (as in EOQ). Instead it is **I_{max} /2**



Non-instantaneous replenishment Finding economic Q

Total annual cost = Annual holding cost + Annual ordering or setup cost

$$C = \frac{I_{max}}{2}(H) + \frac{D}{Q}(S)$$ => $$C = \frac{Q}{2}\left(\frac{p-d}{p}\right)(H) + \frac{D}{Q}(S)$$

D = annual demand
p = production rate
d = demand rate
Q = lot size
H = holding cost per unit
S = (ordering)setup cost per setup

=> **Economic production lot size:**

$$ELS = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$$

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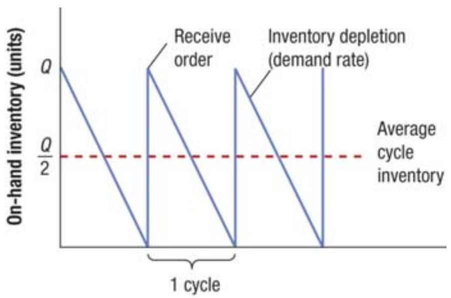
EOQ vs. ELS

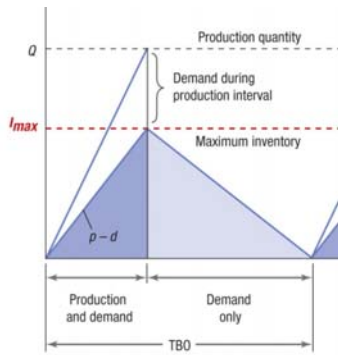
Economic order size for instantaneous replenishment

$$EOQ = \sqrt{\frac{2DS}{H}}$$



Economic production lot size with non-instantaneous replenishment

$$ELS = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$$



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EXAMPLE 2

Non-instantaneous replenishment

- Demand: 30 barrels per day
- Production rate : 190 barrels per day
- Set up cost : $200
- Annual holding cost : $0,21 per barrels
- Operate 350 days per year

Questions:

a) ELS?
b) Annual total cost with ELS?
c) TBO for the ELS?
d) Production time per lot?

A plant manager of a chemical plant must determine the lot size for a particular chemical that has a steady demand of 30 barrels per day. The production rate is 190 barrels per day, annual demand is 10,500 barrels, setup cost is \$200, annual holding cost is \$0.21 per barrel, and the plant operates 350 days per year.

Solution

a. Solving first for the ELS, we get

$$\text{ELS} = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}} = \sqrt{\frac{2(10{,}500)(\$200)}{\$0.21}}\sqrt{\frac{190}{190-30}}$$
$$= 4{,}873.4 \text{ barrels}$$

b. The annual total cost with the ELS is

$$C = \frac{Q}{2}\left(\frac{p-d}{p}\right)(H) + \frac{D}{Q}(S)$$
$$= \frac{4{,}873.4}{2}\left(\frac{190-30}{190}\right)(\$0.21) + \frac{10{,}500}{4{,}873.4}(\$200)$$
$$= \$430.91 + \$430.91 = \$861.82$$

c. Applying the TBO formula (see the Inventory Management chapter) to the ELS, we get

$$\text{TBO}_{\text{ELS}} = \frac{\text{ELS}}{D}(350 \text{ days/year}) = \frac{4{,}873.4}{10{,}500}(350)$$
$$= 162.4, \quad \text{or} \quad 162 \text{ days}$$

d. The production time during each cycle is the lot size divided by the production rate:

$$\frac{\text{ELS}}{p} = \frac{4{,}873.4}{190} = 25.6, \quad \text{or} \quad 26 \text{ days}$$

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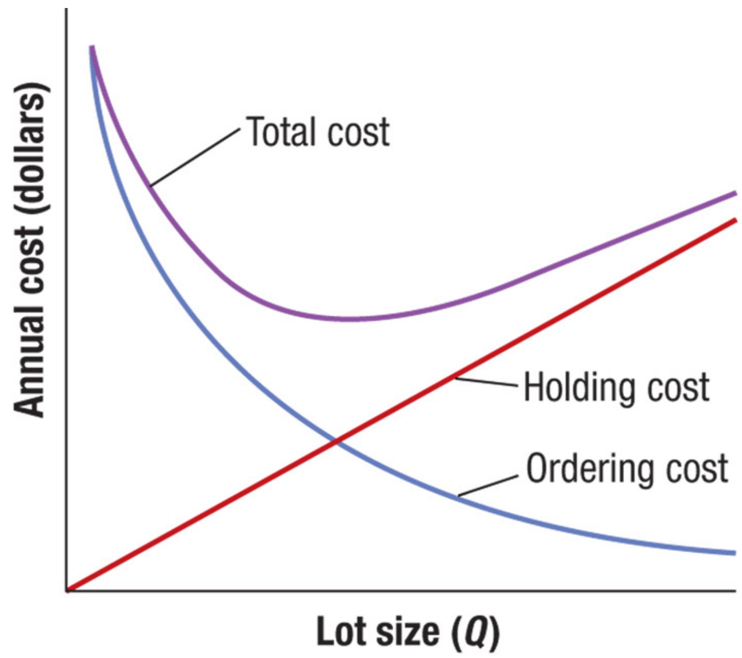
Quantity discounts

Order quantity	Price per unit
0 to 99	$4
100 to 199	$3,5
200 and more	$3



How to find economic order quantity?

Quantity discounts

Total annual cost =Annual holding cost + Annual ordering or setup cost + Annual cost of materials

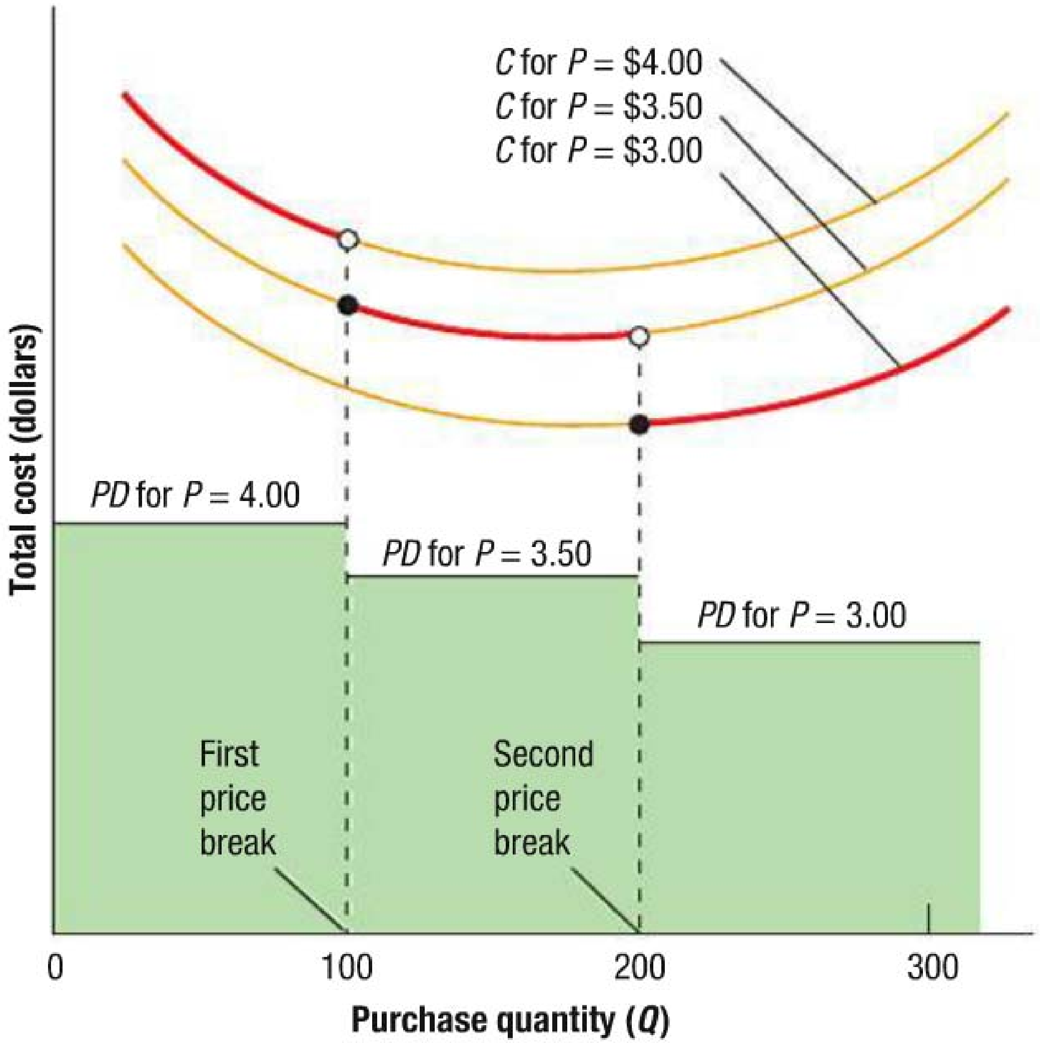
$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + PD$$

where

- Q = lot size
- H = holding cost
- D = annual demand
- d = demand rate
- S = ordering and/or setup cost
- P = price/unit

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + PD$$

Order quantity	Price per unit
0 to 99	$4
100 to 199	$3,5
200 and more	$3



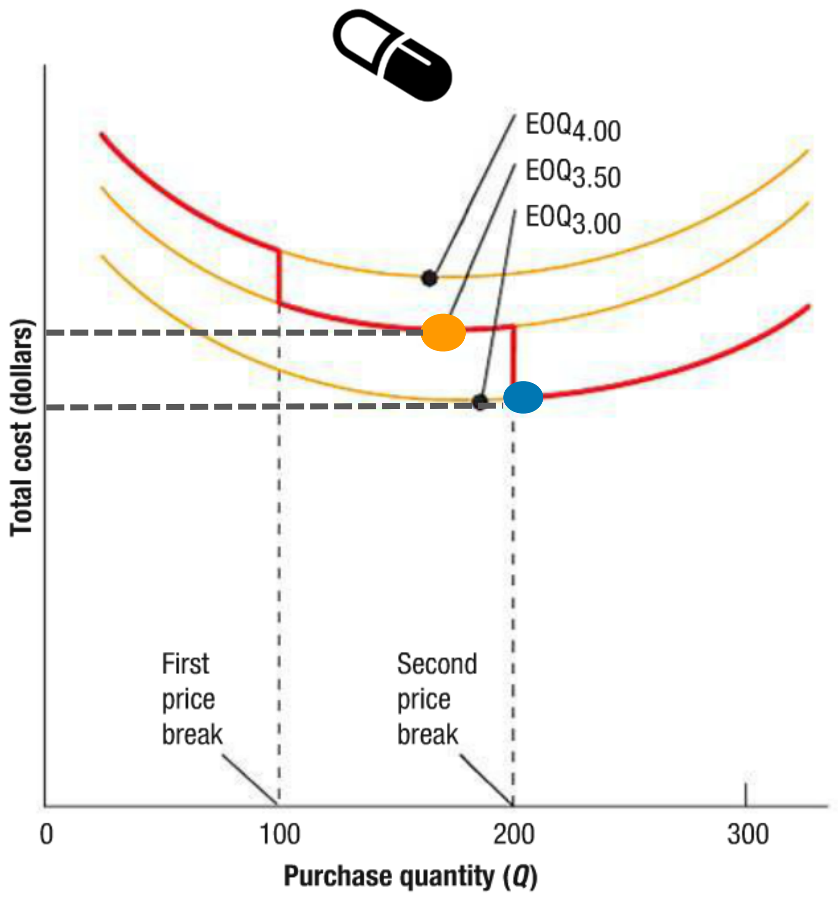
Quantity discounts
How to find economic order quantity?

Step 1:

Calculate EOQ for each price level

Step 2:

- Find feasible EOQ with lowest price level
- Compare total cost
  - at the EOQ
  - at price break quantity at lower price level



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EXAMPLE 3

Quantity discounts

- Annual demand : 936 units
- Order cost : $45 per order
- Annual holding cost : 25% of unit price
- The price schedule in the table

Order Quantity	Price per Unit
0 to 299	$60.00
300 to 499	$58.80
500 or more	$57.00

Question:

Order quantity for minimum total cost?

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Solution

Step1: Calculate EOQ at each price level

$$EOQ_{60.00} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(\$45.00)}{0.25(\$60.00)}} = 75 \text{ units}$$

Feasible

$$EOQ_{58.80} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(\$45.00)}{0.25(\$58.80)}} = 76 \text{ units}$$

$$EOQ_{57.00} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(\$45.00)}{0.25(\$57.00)}} = 77 \text{ units}$$

Order Quantity	Price per Unit
0 to 299	$60.00
300 to 499	$58.80
500 or more	$57.00

Solution

Step2:

- Find feasible EOQ at lowest price
- Compare total cost
  - at the EOQ
  - at price break quantity at lower price level

$$C_{75} = \frac{75}{2}[(0.25)(\$60.00)] + \frac{936}{75}(\$45.00) + \$60.00(936) = \$57,284$$

$$C_{300} = \frac{300}{2}[(0.25)(\$58.80)] + \frac{936}{300}(\$45.00) + \$58.80(936) = \$57,382$$

$$C_{500} = \frac{500}{2}[(0.25)(\$57.00)] + \frac{936}{500}(\$45.00) + \$57.00(936) = \underline{\$56,999}$$

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One period decisions

One of the dilemmas facing many retailers is how to **handle seasonal goods**. Often, they cannot be sold at full markup the next year (or other periods) because of changes in styles. This type of situation is often called the newsboy problem.

One period decisions - how to find economic order quantity?

Step 1: List different demand levels and probabilities

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

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One period decisions - how to find economic order quantity?

Step 2: Develop a payoff table as below

Order quantity	Demand 10	Demand 20	Demand 30	Demand 40	Demand 50
10	green	green	green	green	green
20	pink	green	green	green	green
30	pink	pink	green	green	green
40	pink	pink	pink	green	green
50	pink	pink	pink	pink	green

Green: You sell them all

Payoff = pQ

Pink: You don't sell out

Payoff = $pD - l(Q - D)$

p: profit per unit
l : loss per unit

One period decisions - how to find economic order quantity?

Step 2: Develop a payoff table as below

Order quantity	Demand				
	10	20	30	40	50
10	100	100			
20					
30					
40	-50				
50					

p: profit per unit = 10$
l : loss per unit = 5$

You sell them all

Payoff = pQ

You don't sell out

Payoff = $pD - l(Q - D)$

= 10x10 – 5(40-10)
= - 50

One period decisions - how to find economic order quantity?

Step 2: Develop a payoff table as below

Order quantity	Demand				
	10	20	30	40	50
10	100	100	100	100	100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

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One period decisions - how to find economic order quantity?

Step 3: Calculate the expected payoff of each *Q*

Order quantity \ Demand	10	20	30	40	50
10	100	100	100	100	100
20	50	200	200	200	200
30	0	150	300	300	300
40	-50	100	250	400	400
50	-100	50	200	350	500

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

$$E(X) = \sum x_i p_i$$

E(Payoff at Q=30) = 0*0,2+150*0,3+300*0,3+300*0,1+300*0,1

= 195

One period decisions - how to find economic order quantity?

Step 4: Choose the order quantity *Q* with the highest expected payoff.

Order quantity	Demand					Expected payoff
	10	20	30	40	50	
10	100	100	100	100	100	100
20	50	200	200	200	200	170
30	0	150	300	300	300	195
40	-50	100	250	400	400	175
50	-100	50	200	350	500	140

EXAMPLE 4

One period decisions

Swell Productions is sponsoring an outdoor conclave for owners of collectible and classic Fords. The concession stand in the T-Bird area will sell clothing such as T-shirts and official Thunderbird racing jerseys. Jerseys are purchased from Columbia Products for $40 each and are sold during the event for $75 each. If any jerseys are left over, they can be returned to Columbia for a refund of $30 each. Jersey sales depend on the weather, attendance, and other variables. The following table shows the probability of various sales quantities. How many jerseys should Swell Productions order from Columbia for this one-time event?

Sales Quantity	Probability	Quantity Sales	Probability
100	0.05	400	0.34
200	0.11	500	0.11
300	0.34	600	0.05

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Sales Quantity	Probability	Quantity Sales	Probability
100	0.05	400	0.34
200	0.11	500	0.11
300	0.34	600	0.05

SOLUTION

Table D.1 is the payoff table that describes this one-period inventory decision. The upper right portion of the table shows the payoffs when the demand, *D*, is greater than or equal to the order quantity, *Q*. The payoff is equal to the per-unit profit (the difference between price and cost) multiplied by the order quantity. For example, when the order quantity is 100 and the demand is 200,

$$\text{Payoff} = (p - c)Q = (\$75 - \$40)100 = \$3{,}500$$

TABLE D.1 | PAYOFFS

Q	Demand, *D* 100	200	300	400	500	600	Expected Payoff
100	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500
200	$2,500	$7,000	$7,000	$7,000	$7,000	$7,000	$6,775
300	$1,500	$6,000	$10,500	$10,500	$10,500	$10,500	$9,555
400	$500	$5,000	$9,500	$14,000	$14,000	$14,000	$10,805
500	($500)	$4,000	$8,500	$13,000	$17,500	$17,500	$10,525
600	($1,500)	$3,000	$7,000	$12,000	$16,500	$21,000	$9,750

Demand exceeds supply => Case 1

Supply exceeds demand => Case 2

The lower-left portion of the payoff table shows the payoffs when the order quantity exceeds the demand. Here the payoff is the profit from sales, pD, minus the loss associated with returning overstock, $l(Q - D)$, where l is the difference between the cost and the amount refunded for each jersey returned and $Q - D$ is the number of jerseys returned. For example, when the order quantity is 500 and the demand is 200,

$$\text{Payoff} = pD - l(Q - D) = (\$75 - \$40)200 - (\$40 - \$30)(500 - 200) = \$4,000$$

The highest expected payoff occurs when 400 jerseys are ordered:

$$\text{Expected payoff}_{400} = (\$500 \times 0.05) + (\$5,000 \times 0.11) + (\$9,500 \times 0.34) + (\$14,000 \times 0.34) + (\$14,000 \times 0.11) + (\$14,000 \times 0.05) = \$10,805$$

Sales Quantity	Probability	Quantity Sales	Probability
100	0.05	400	0.34
200	0.11	500	0.11
300	0.34	600	0.05

TABLE D.1 | PAYOFFS

Q	Demand, *D* 100	200	300	400	500	600	Expected Payoff
100	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500
200	$2,500	$7,000	$7,000	$7,000	$7,000	$7,000	$6,775
300	$1,500	$6,000	$10,500	$10,500	$10,500	$10,500	$9,555
400	$500	$5,000	$9,500	$14,000	$14,000	$14,000	$10,805
500	($500)	$4,000	$8,500	$13,000	$17,500	$17,500	$10,525
600	($1,500)	$3,000	$7,000	$12,000	$16,500	$21,000	$9,750

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Relevant book chapters

- Chapter: Managing inventories
- Supplement C: Special inventory models